Stochastic modeling of the oceanic mesoscale eddies

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Long Li[1], Bruno Deremble[2], Noé Lahaye[1] and Etienne Mémin[1]

[1] *Inria Rennes Bretagne Atlantique, France*

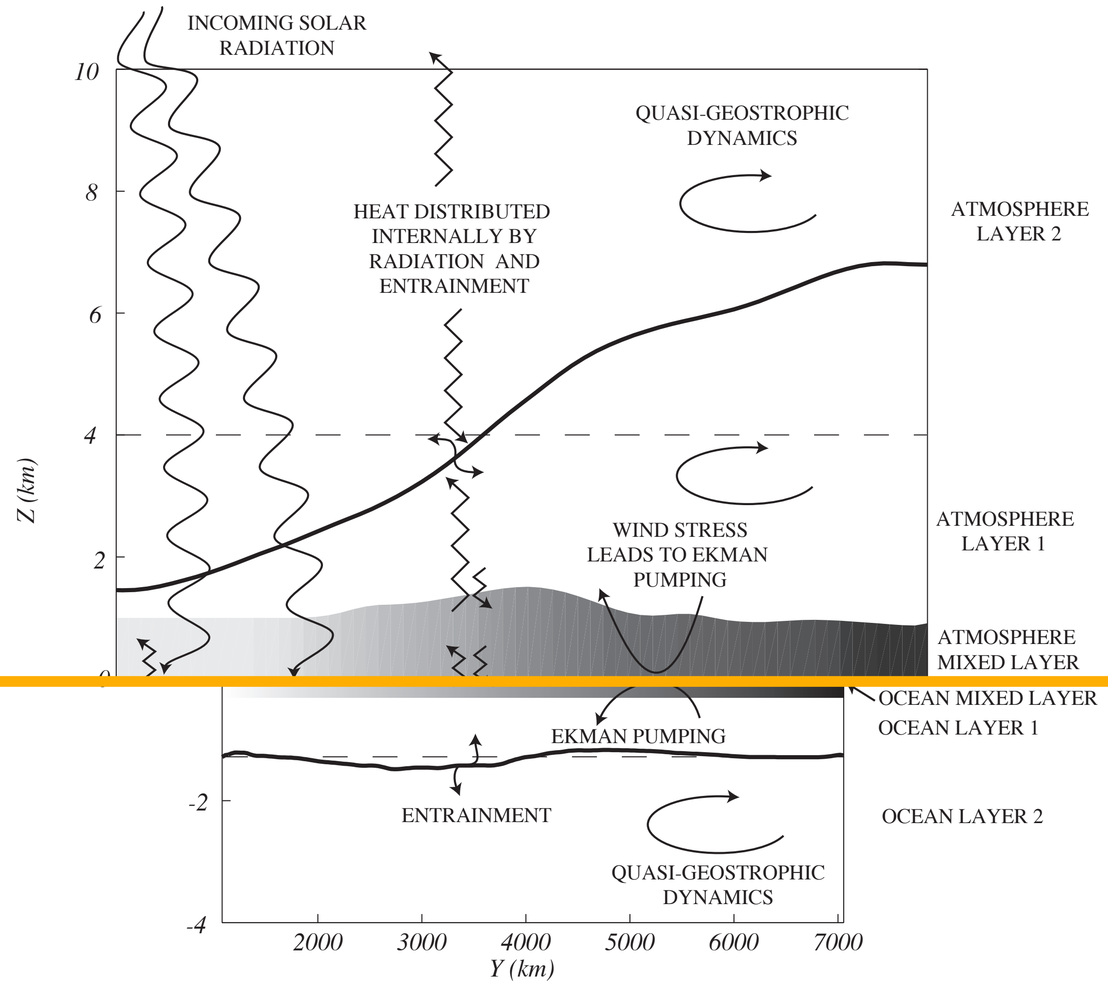
[2] *Institut des Géosciences de l'Environnement, France*

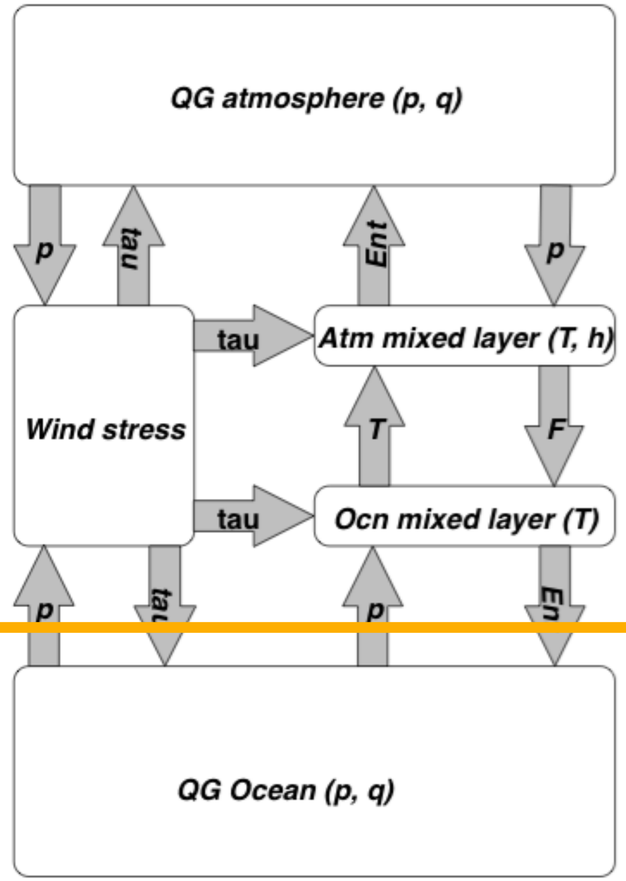
Motivations

I. Can LU model better represent the mesoscale eddies effect on the large-scale circulation?

II. Can LU model improve the internal variability of coarse-resolution ocean models?

Dynamical core





Quasi-Geostrophic Coupled Model (Q-GCM) [http://q-gcm.org]

Deterministic QG equations

- **Evolution of k-th layer potential vorticity (PV)**

$$\partial_t q_k = -\boldsymbol{\nabla}\cdot(\boldsymbol{u}_k q_k) + \frac{A_2}{f_0}\nabla^4 p_k - \frac{A_4}{f_0}\nabla^6 p_k - \frac{f_0}{H_k}(w_k - w_{k-1})$$

- **Update k-th layer pressure, height and velocity**

$$q_k = \frac{1}{f_0}\nabla^4 p_k + \beta(y - y_0) + \frac{f_0}{H_k}(\eta_k - \eta_{k-1})$$

$$\eta_k = \frac{p_{k+1} - p_k}{g'_k}, \qquad g'_k = \frac{g(\rho_{k+1} - \rho_k)}{\rho_0}$$

$$\boldsymbol{u}_k = \frac{1}{f_0}\boldsymbol{\nabla}^\perp p_k$$

Exp.1

- **Evolution of mixed layer temperature (or SST)**

$$\boldsymbol{u}_m = \boldsymbol{u}_1 + \frac{\boldsymbol{\tau}^\perp}{f_0 H_m}$$

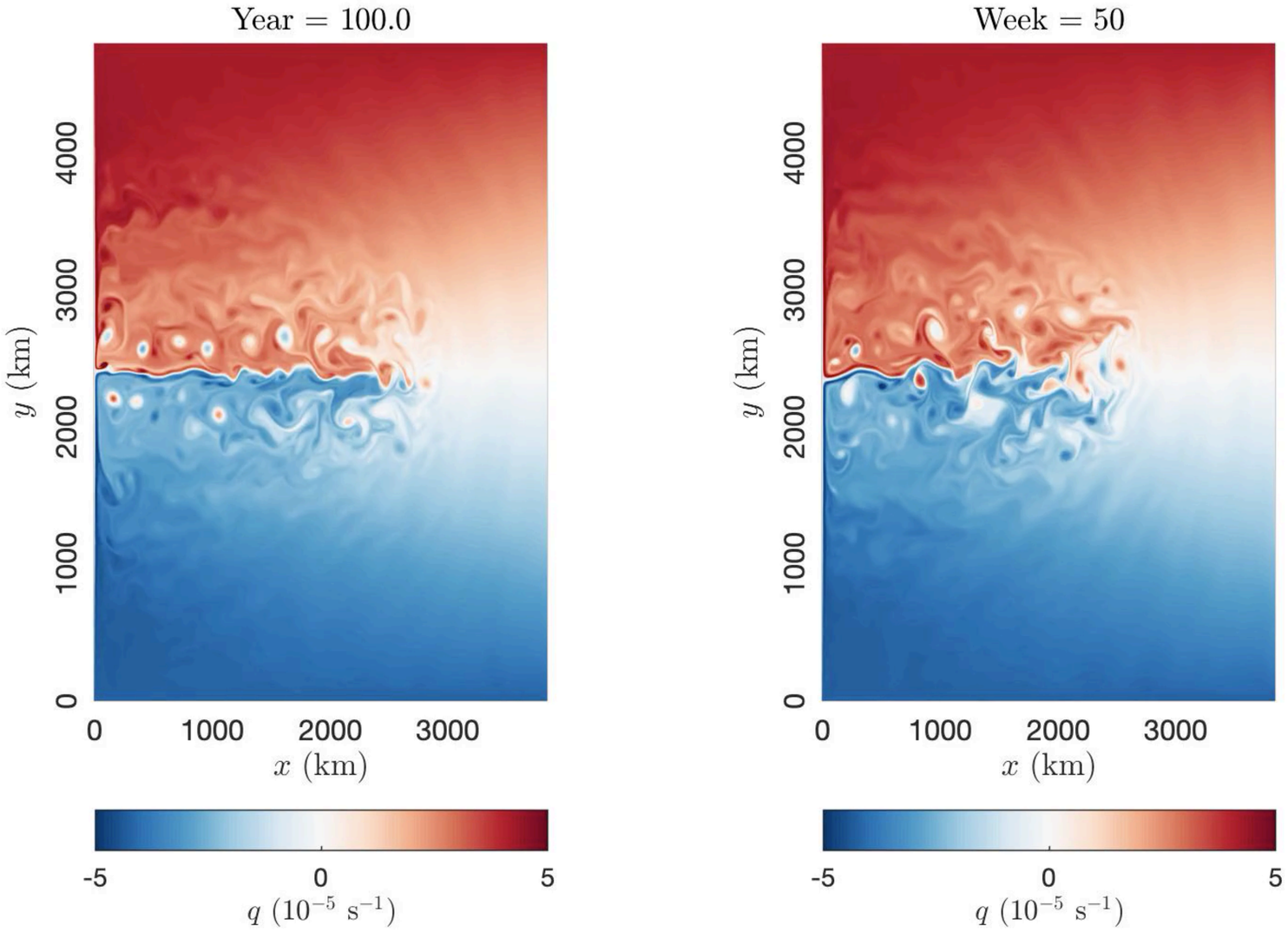
$$\partial_t T_m = -\boldsymbol{\nabla}\cdot(\boldsymbol{u}_m T_m) + K_2\nabla^2 T_m - K_4\nabla^4 T_m + w_0\frac{T_1 + T_m}{2H_m} - \mathrm{F}_m$$

Exp.2

$$\begin{bmatrix} w_0 = \frac{1}{f_0}\boldsymbol{\nabla}\times\boldsymbol{\tau} \\ \\ w_N = \frac{\delta_{\mathrm{ek}}}{2f_0}\nabla^2 p_N \end{bmatrix}$$

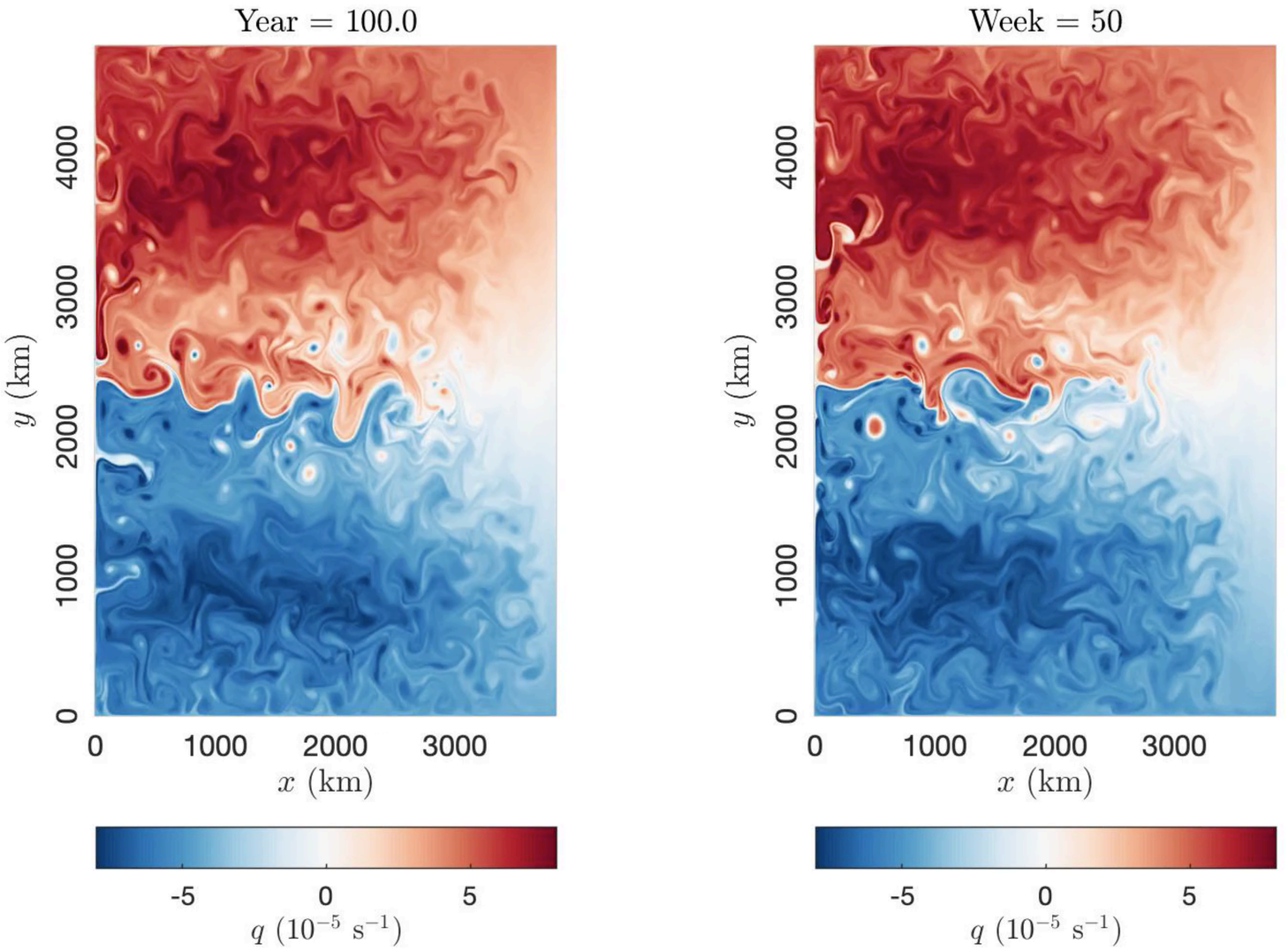
$$w_1 = -\frac{\Delta_m T}{2\Delta_1 T} w_0$$

Eddy-resolving simulations



Upper-layer PV ($\Delta x = 5$ km, $L_d = [39,22]$ km) **of Exp.1** (without SST)

Eddy-resolving simulations



Upper-layer PV ($\Delta x = 5$ km, $L_d = [39,22]$ km) **of Exp.2** (with SST)

Stochastic QG equations

- **k-th layer eddy velocity noise** (m/s) $\quad \boldsymbol{\sigma}_k \dot{\boldsymbol{B}}_t$

- **k-th layer variance tensor** (m^2/s^2) $\quad \dfrac{\boldsymbol{a}_k}{\mathrm{d}t} = \mathbb{E}\Big[\big(\boldsymbol{\sigma}_k \dot{\boldsymbol{B}}_t - \mathbb{E}[\boldsymbol{\sigma}_k \dot{\boldsymbol{B}}_t]\big)\big(\boldsymbol{\sigma}_k \dot{\boldsymbol{B}}_t - \mathbb{E}[\boldsymbol{\sigma}_k \dot{\boldsymbol{B}}_t]\big)^T\Big]$

- **Evolution of k-th layer PV** $\quad \partial_t q_k = -\boldsymbol{\nabla}\cdot(\boldsymbol{u}_k q_k) + \dfrac{A_2}{f_0}\nabla^4 p_k - \dfrac{A_4}{f_0}\nabla^6 p_k - \dfrac{f_0}{H_k}(w_k - w_{k-1}) + \boldsymbol{\nabla}\cdot\boldsymbol{F}_k$

$$\boldsymbol{F}_k = -\big(\boldsymbol{\sigma}_k \dot{\boldsymbol{B}}_t - \frac{1}{2}\boldsymbol{\nabla}\cdot\boldsymbol{a}_k\big) q_k + \frac{1}{2}\boldsymbol{a}_k \boldsymbol{\nabla} q_k - \big(\boldsymbol{u}_k \cdot \boldsymbol{\nabla}^{\perp}\boldsymbol{\sigma}_k \dot{\boldsymbol{B}}_t + \boldsymbol{a}_k \boldsymbol{\nabla} f\big) + \frac{1}{2}\sum_{i=1,2} \partial^{\perp}_{x_i} \boldsymbol{a}_k \boldsymbol{\nabla} u^i_k$$

Advection flux **Diffusion flux** **Sources flux** **Sinks flux**

- **Evolution of SST** $\quad \partial_t T_m = -\boldsymbol{\nabla}\cdot(\boldsymbol{u}_m T_m) + K_2\nabla^2 T_m - K_4\nabla^4 T_m + w_0 \dfrac{T_1 + T_m}{2H_m} - \mathrm{F}_m + \boldsymbol{\nabla}\cdot\boldsymbol{F}_m$

$$\boldsymbol{F}_m = -\big(\boldsymbol{\sigma}_1 \dot{\boldsymbol{B}}_t - \frac{1}{2}\boldsymbol{\nabla}\cdot\boldsymbol{a}_1\big) T_m + \frac{1}{2}\boldsymbol{a}_1 \boldsymbol{\nabla} T_m$$

Data-driven modeling

(i) LU-POD

$$\boldsymbol{u}_k^{\mathrm{HR}} - \overline{\boldsymbol{u}_k^{\mathrm{HR}}} = \mathcal{F}\boldsymbol{u}_k^{\mathrm{HR}} + (1-\mathcal{F})\boldsymbol{u}_k^{\mathrm{HR}} - \overline{\mathcal{F}\boldsymbol{u}_k^{\mathrm{HR}} + (1-\mathcal{F})\boldsymbol{u}_k^{\mathrm{HR}}}$$

$$= (\boldsymbol{u}_k^{\mathrm{LR}} - \overline{\boldsymbol{u}_k^{\mathrm{LR}}}) + (1-\mathcal{F})\boldsymbol{u}_k^{\mathrm{HR}} - \overline{(1-\mathcal{F})\boldsymbol{u}_k^{\mathrm{HR}}}$$

POD

$$\{\boldsymbol{\phi}_{k,n}\}_{n=1,\ldots,N_t}$$

$$\boldsymbol{u}_k^{\star} = -\mathcal{F}\overline{(1-\mathcal{F})\boldsymbol{u}_k^{\mathrm{HR}}}$$

$$\left[\boldsymbol{u}_k^{\star} \neq 0 \text{ if } \mathcal{F}^2 \neq \mathcal{F}\right]$$

$$\boldsymbol{\sigma}_k \dot{\boldsymbol{B}}_t = \sum_{n=M_0}^{M_1} \boldsymbol{\phi}_{k,n}\xi_n + \boldsymbol{u}_k^{\star}$$

$$\frac{\boldsymbol{a}_k}{\mathrm{d}t} = \sum_{n=M_0}^{M_1} \boldsymbol{\phi}_{k,n}\boldsymbol{\phi}_{k,n}^T$$

$\xi_n \sim \mathcal{N}(0,1)$

$(M_0 < M_1 < N_t)$

Data-driven modeling

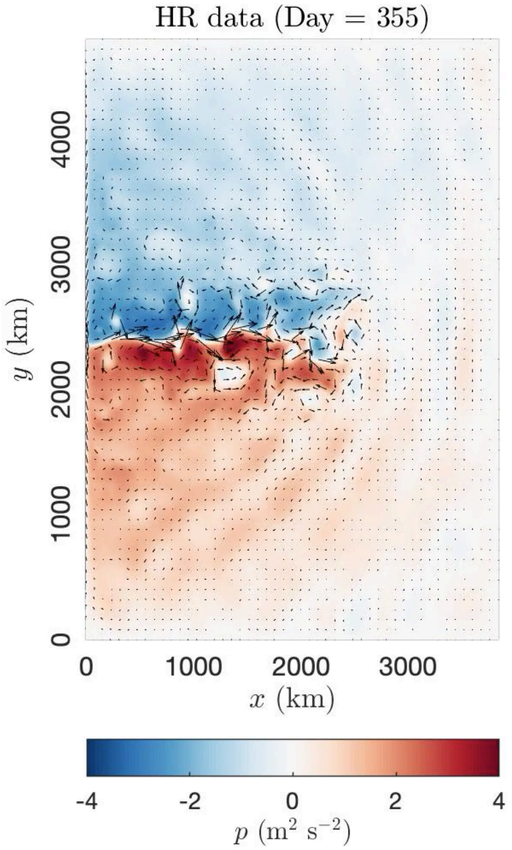
(ii) LU-POD-P

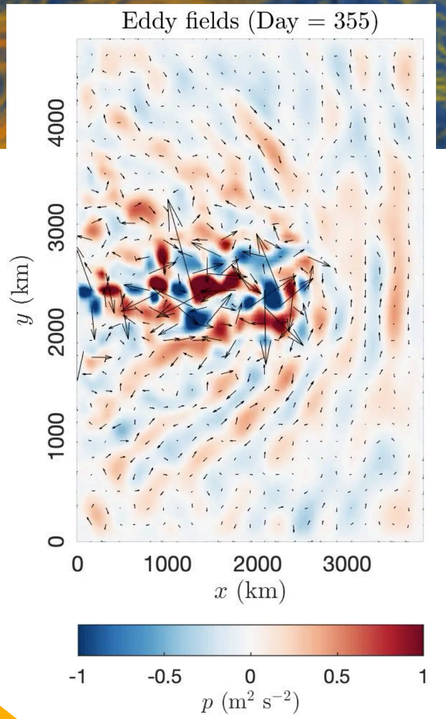
$$\mathbf{P}_k = \mathbf{I} - \frac{\boldsymbol{\nabla}\theta_k(\boldsymbol{\nabla}\theta_k)^T}{\|\boldsymbol{\nabla}\theta_k\|^2}, \qquad \theta_k = \frac{f_0}{H_k}(\eta_k - \eta_{k-1})$$

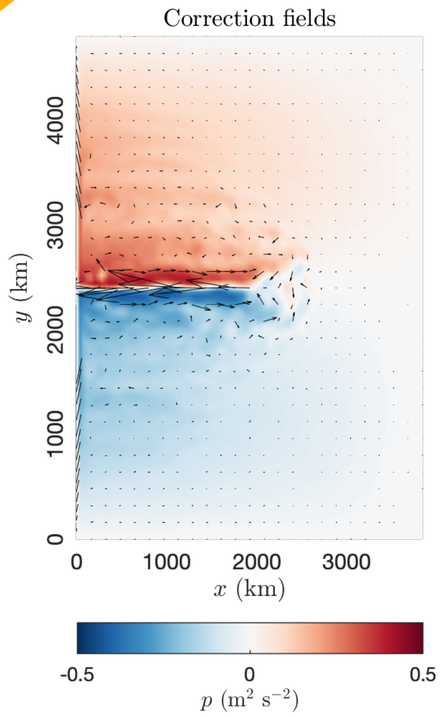
$$\widetilde{\boldsymbol{\sigma}}_k \dot{\boldsymbol{B}}_t = \mathbf{P}_k \boldsymbol{\sigma}_k \dot{\boldsymbol{B}}_t, \qquad \widetilde{\boldsymbol{a}}_k = \mathbf{P}_k \, \boldsymbol{a}_k \, \mathbf{P}_k^T$$

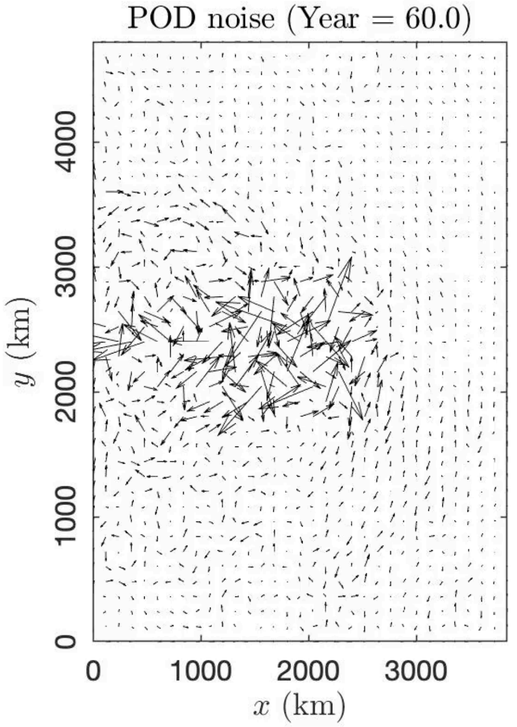
LU-POD

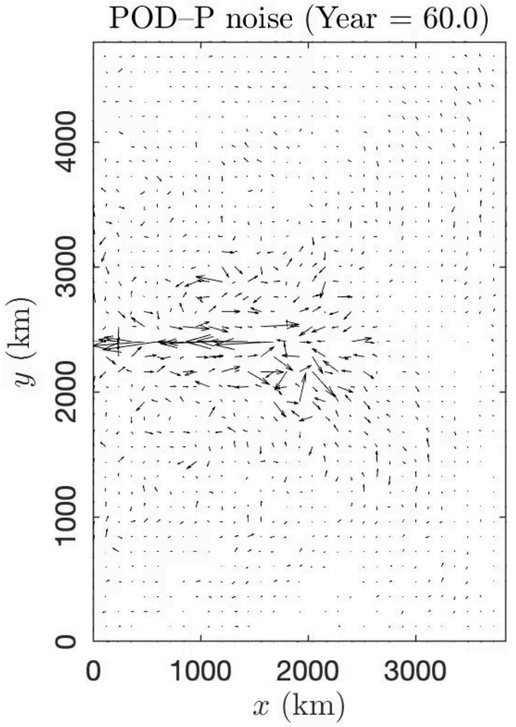
Illustration of noises





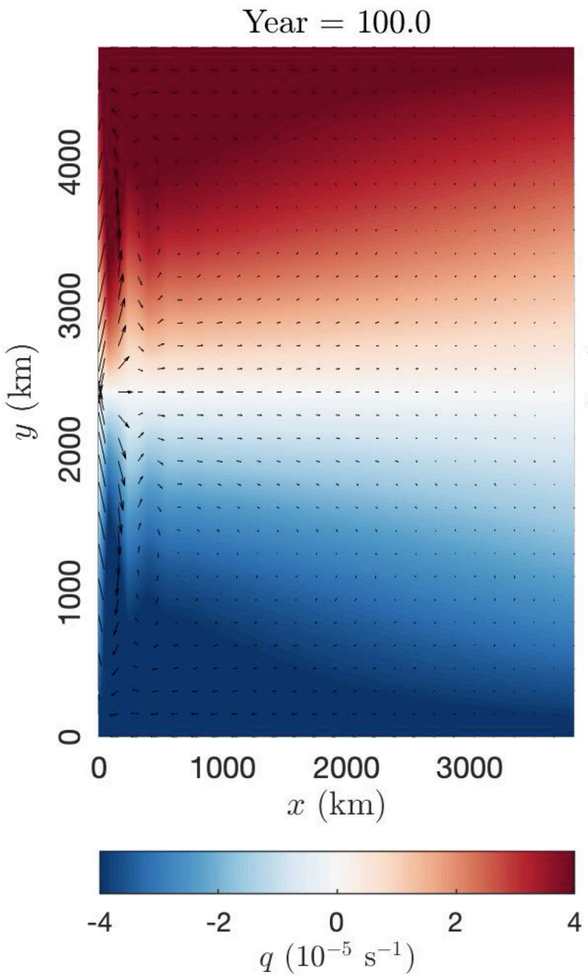




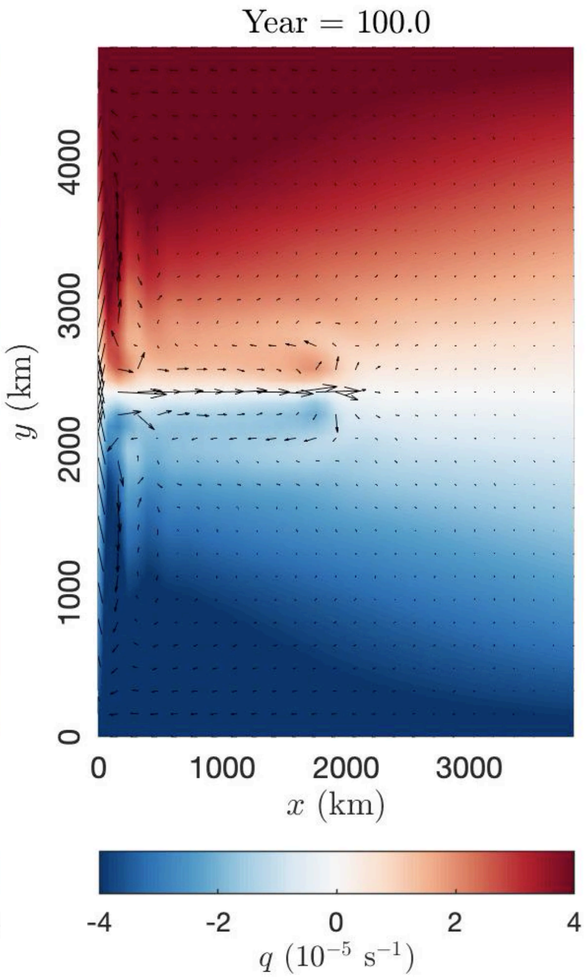


Coarse-resolution simulations

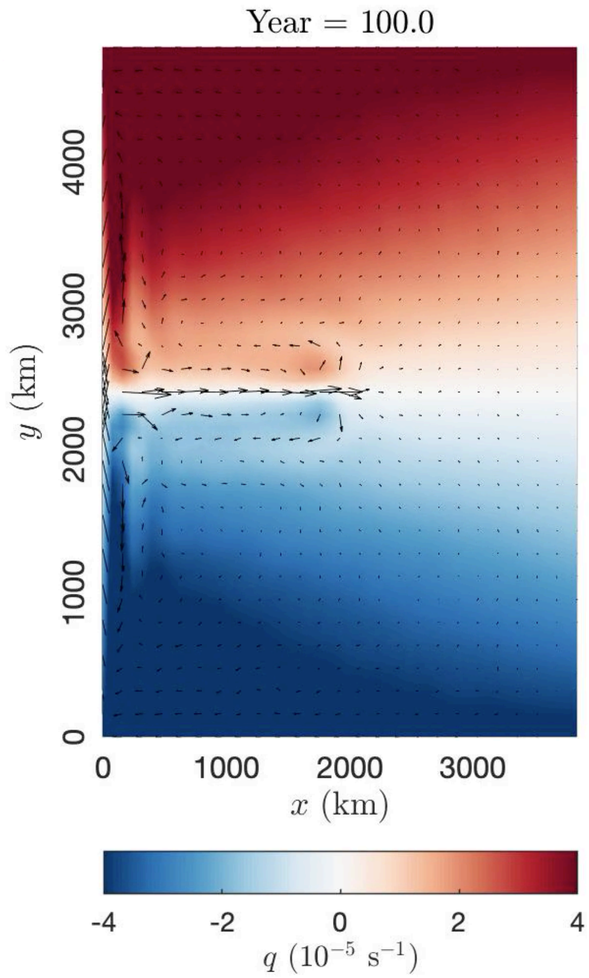
LR

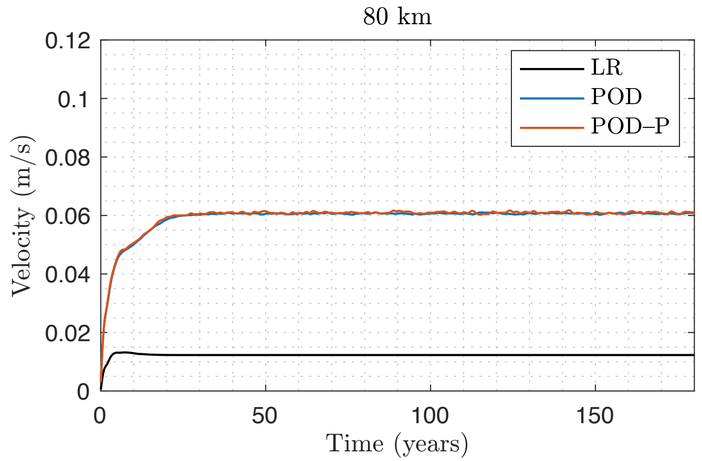


LU-POD



LU-POD-P



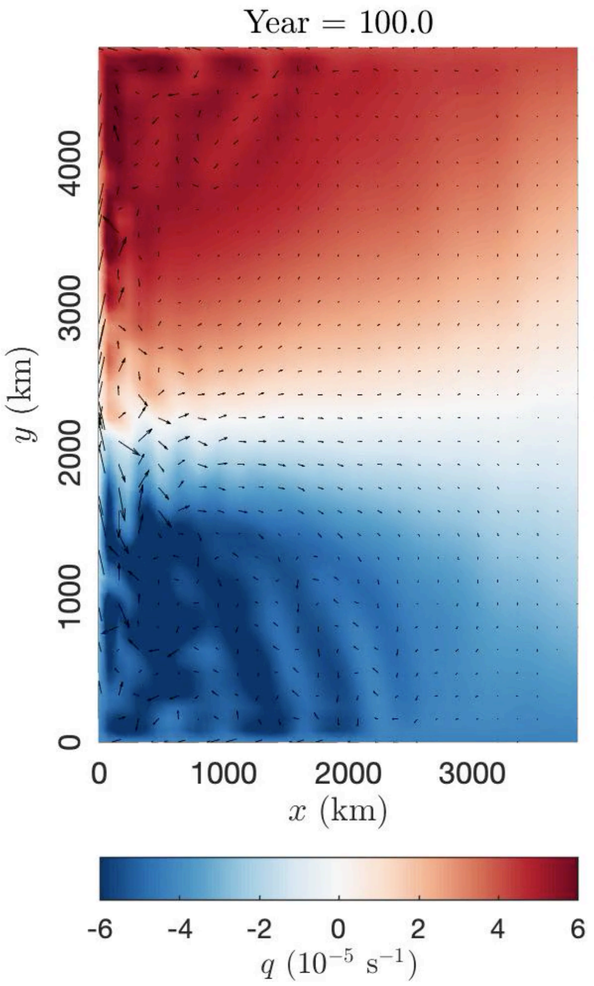


Comparison of jet velocity magnitude

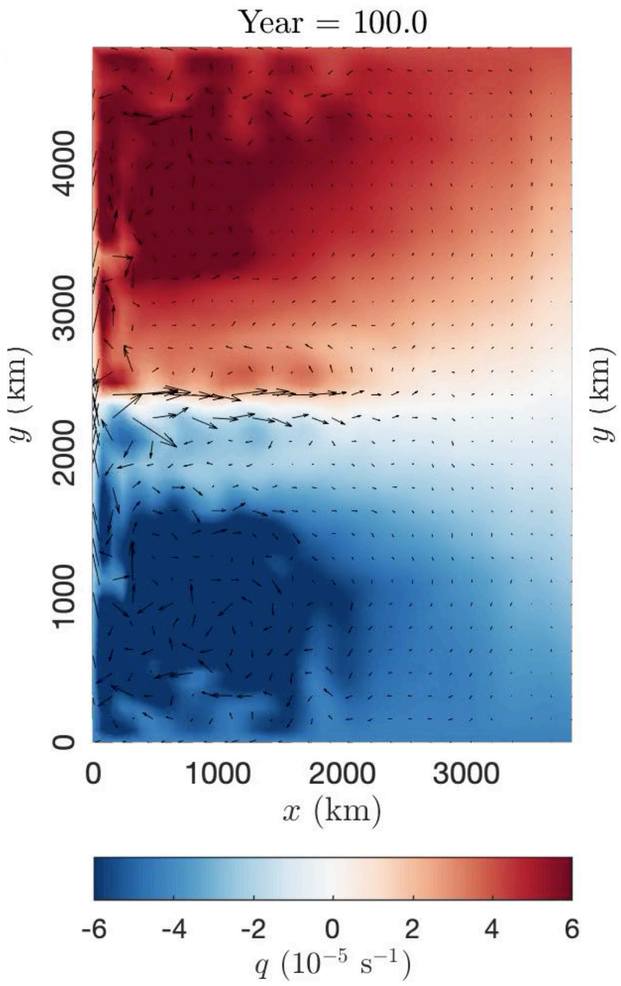
Upper-layer PV ($\Delta x = 80$ km, $L_d = [39,22]$ km) **of Exp.1** (without SST)

Coarse-resolution simulations

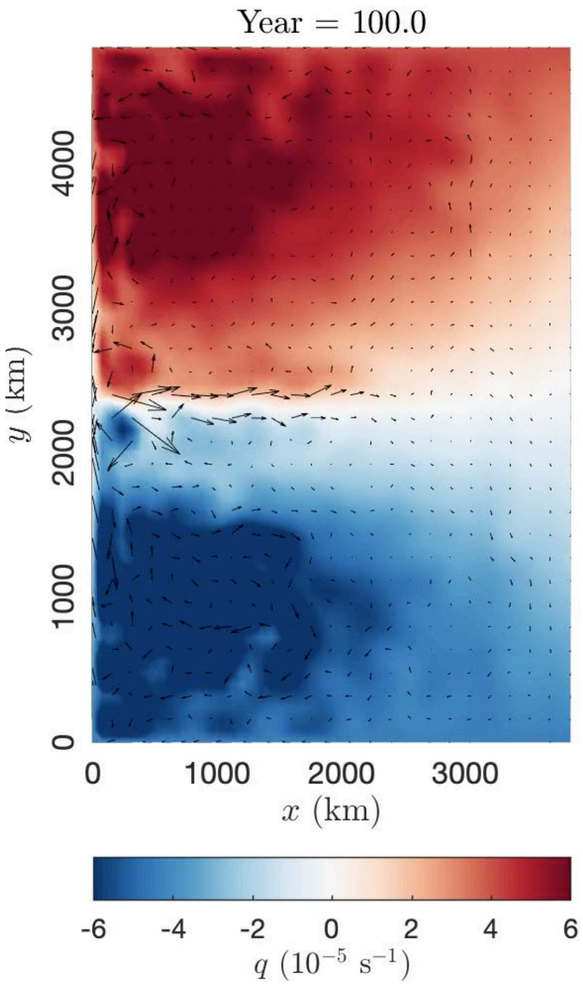
LR

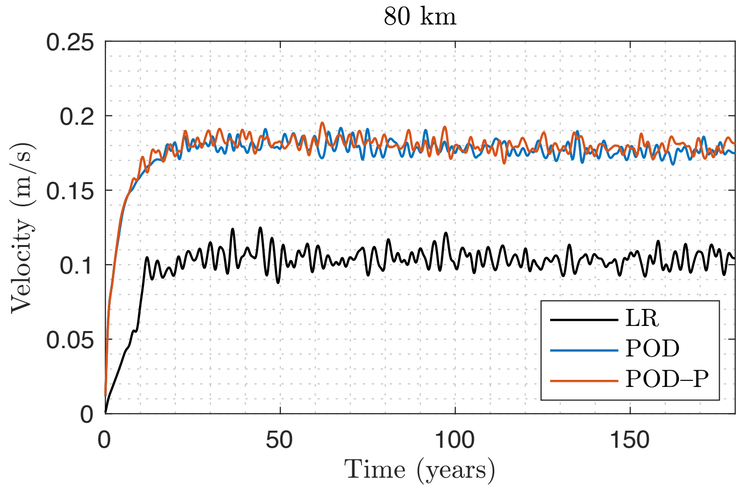


LU-POD



LU-POD-P



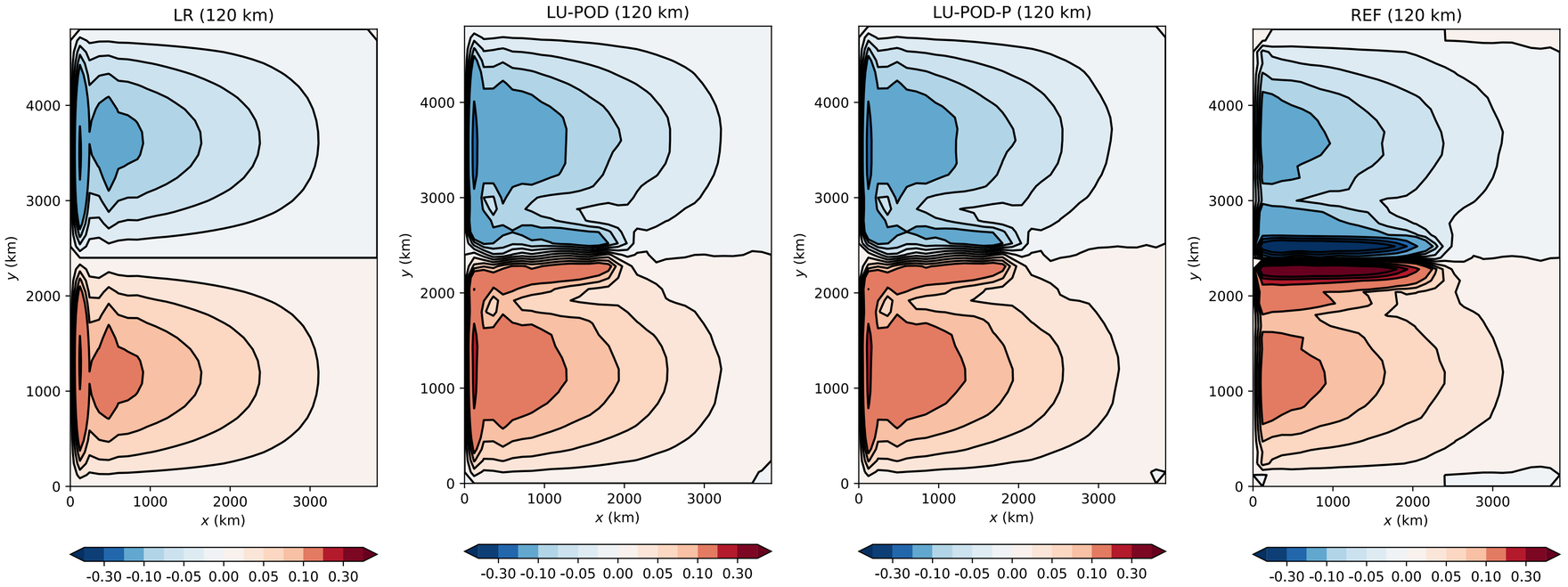


Comparison of jet velocity magnitude

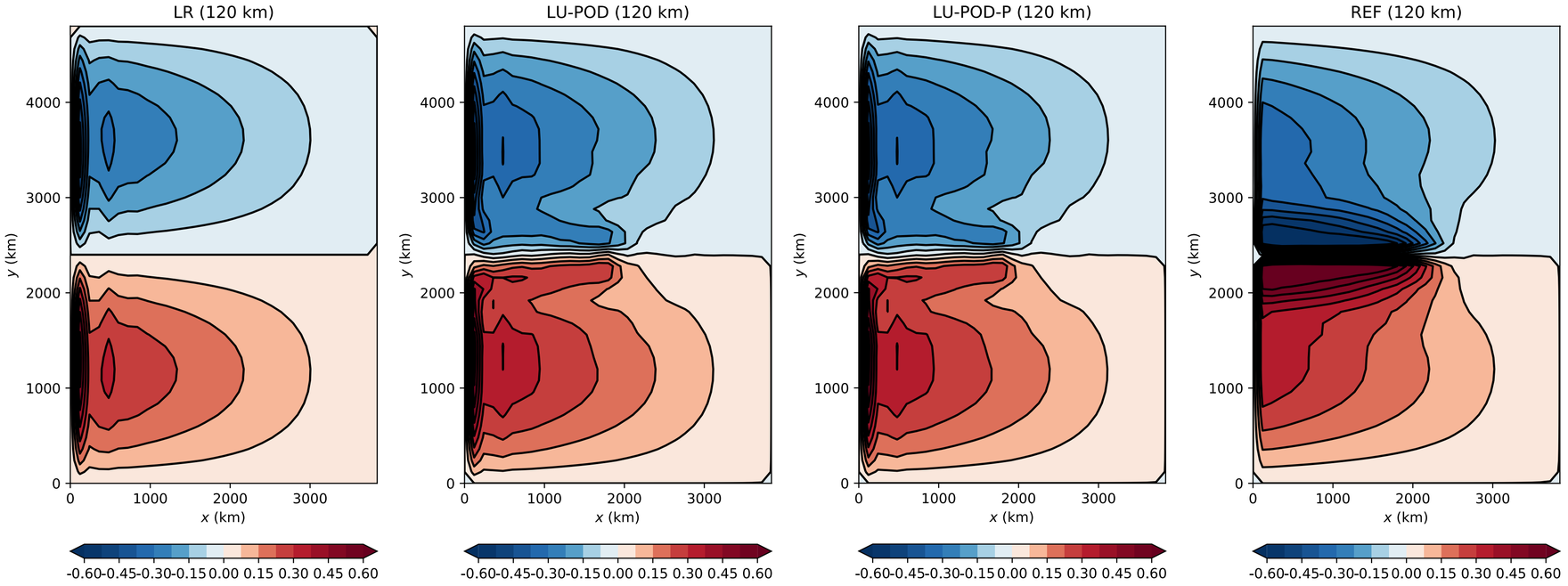
Upper-layer PV ($\Delta x = 80$ km, $L_d = [39,22]$ km) **of Exp.2** (with SST)

Statistical prediction

Barotropic mode



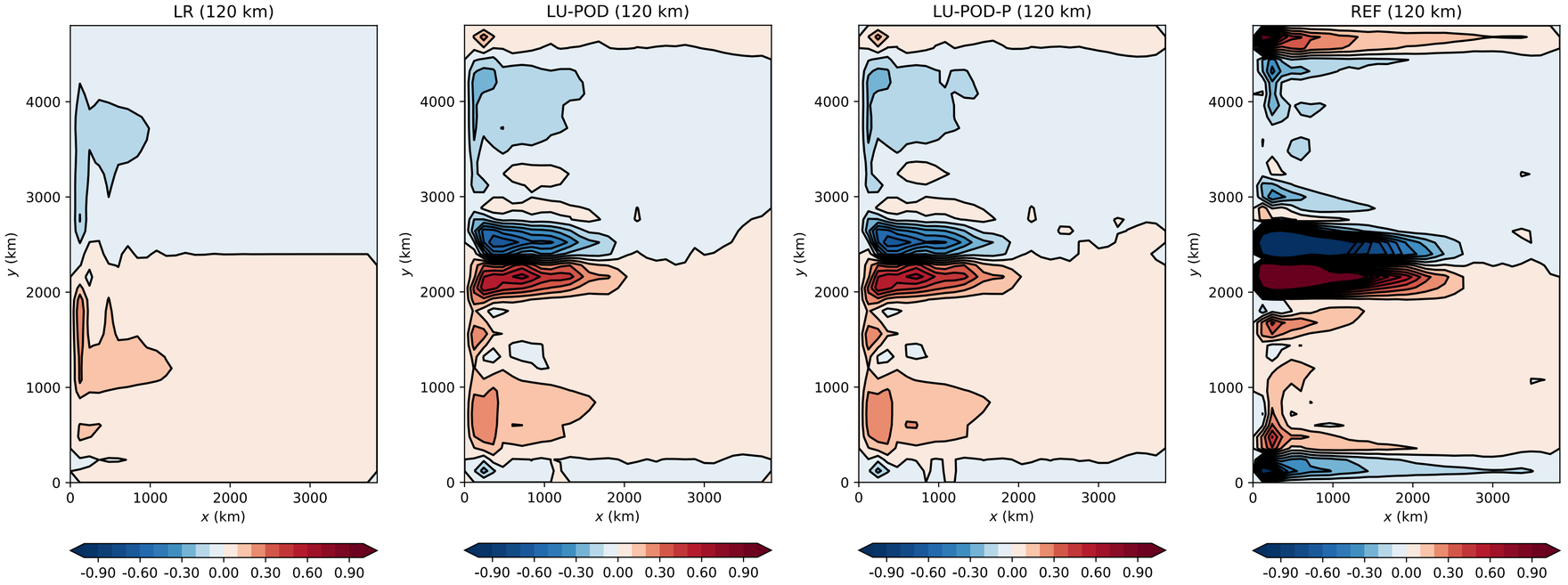
1st Baroclinic mode



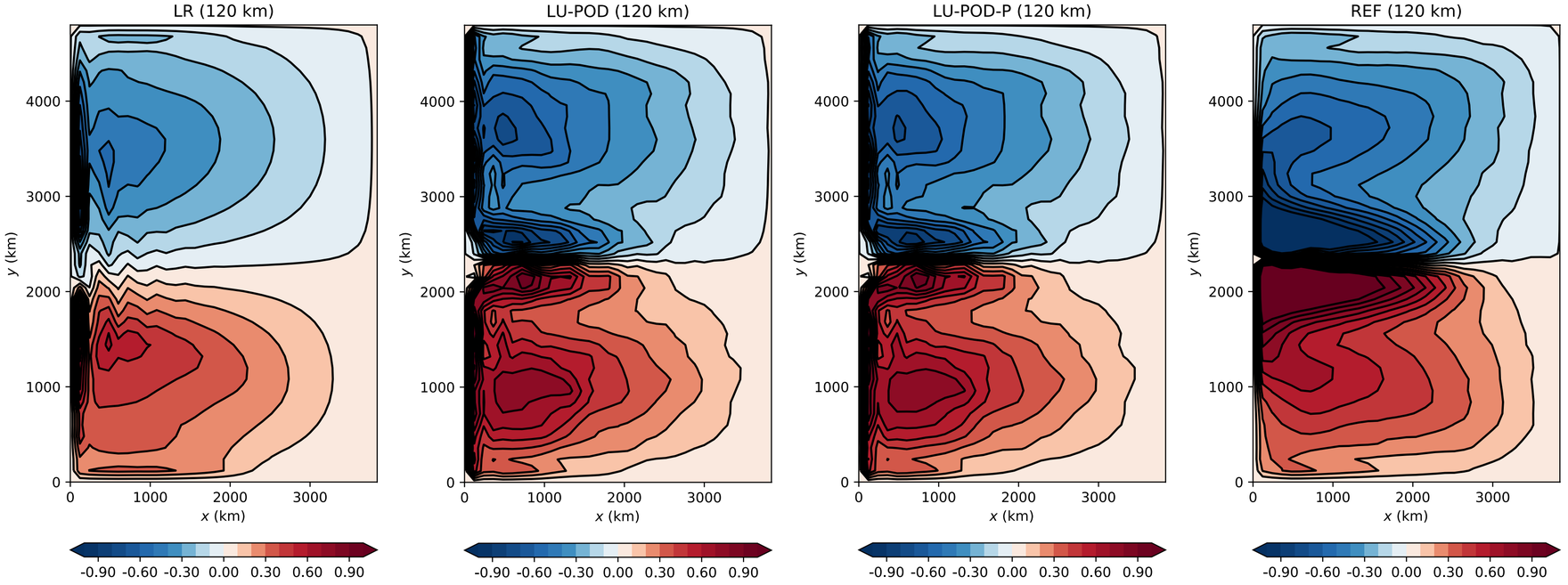
Comparison of mean (60-180 yrs) contour of pressure for coarse-model (120 km) simulations (Exp.1)

Statistical prediction

Barotropic mode



1st Baroclinic mode



Comparison of mean (60-180 yrs) contour of pressure for coarse-model (120 km) simulations (Exp.2)

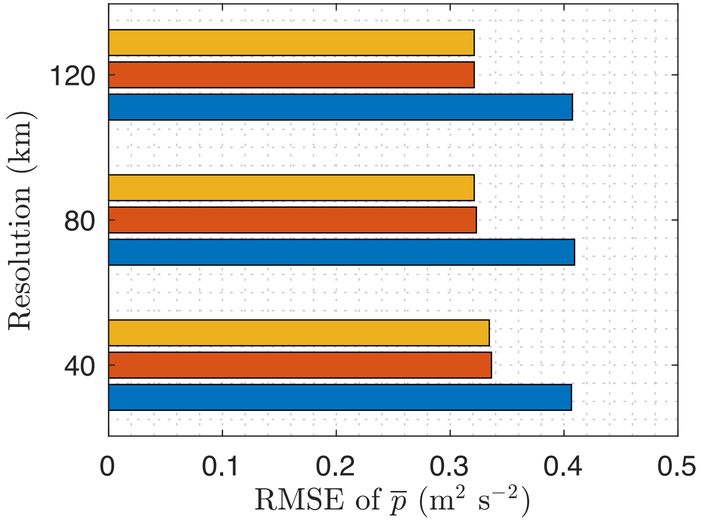
Measures of statistics

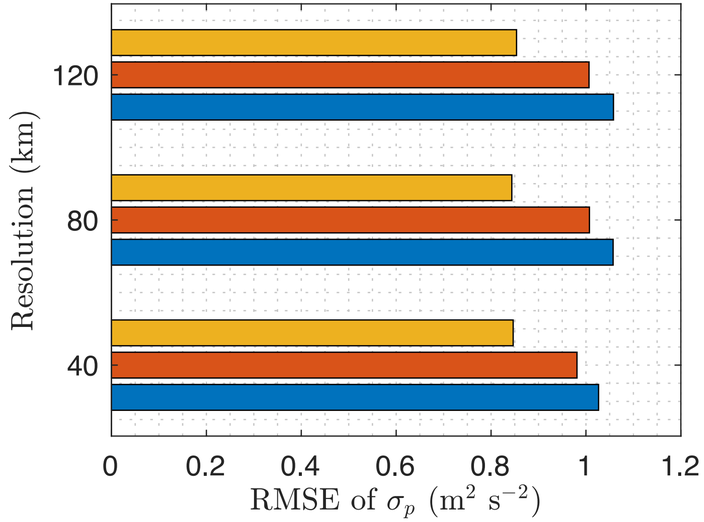
Measures	Values	Accuracy
$\text{RMSE of } \overline{p}_{\text{MOD}} = \|\overline{p}_{\text{MOD}} - \overline{p}_{\text{REF}}\|_A$	↘	+
$\text{RMSE of } \sigma_{\text{MOD}} = \|\sigma_{\text{MOD}} - \sigma_{\text{REF}}\|_A, \quad \sigma = \overline{(p-\overline{p})^2}$	↘	+
$\text{PC} = \dfrac{\|\sigma_{\text{REF}}\sigma_{\text{MOD}}\|_A^2}{\|\sigma_{\text{REF}}^2\|_A \|\sigma_{\text{MOD}}^2\|_A}$	↗	+
$\text{Dispersion} = \dfrac{1}{\|A\|}\int\int \left(\dfrac{\sigma_{\text{REF}}^2}{\sigma_{\text{MOD}}^2} - 1 - \log\left(\dfrac{\sigma_{\text{REF}}^2}{\sigma_{\text{MOD}}^2}\right)\right) \mathrm{d}A$	↘	+
$\text{GRE} = \dfrac{1}{2}\left\|\dfrac{\overline{p}_{\text{REF}} - \overline{p}_{\text{MOD}}}{\sigma_{\text{REF}}}\right\|_A^2 + \dfrac{1}{2}\ \text{Dispersion}$	↘	+

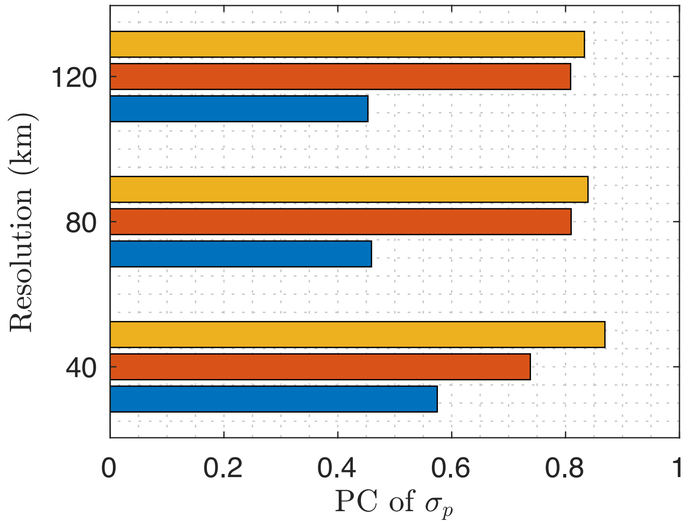
$$\|f\|_A = \left(\frac{1}{|A|}\int\int f^2 \,\mathrm{d}A\right)^{1/2}$$

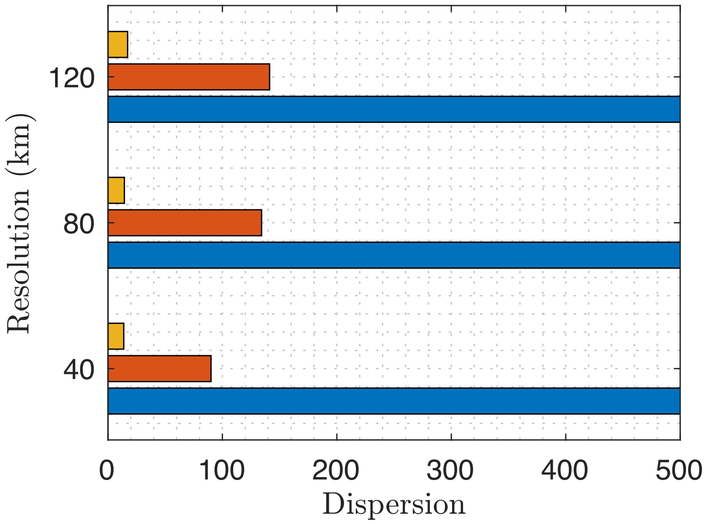
Statistical prediction

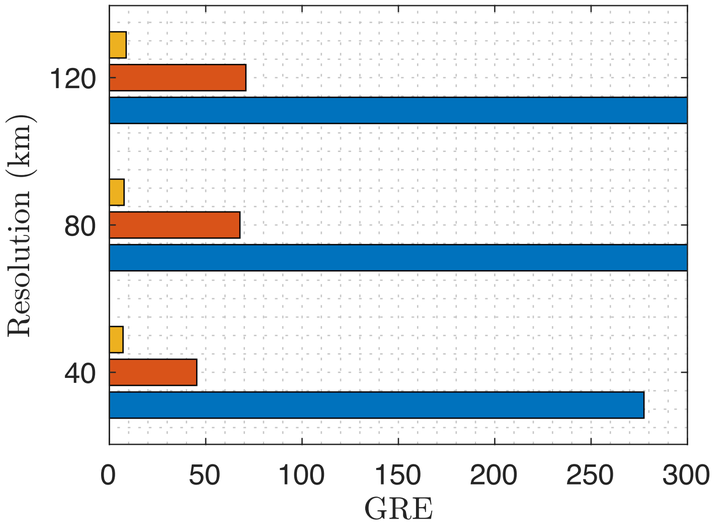
Comparison of statistical measures (integrated vertically) for pressures provided by different coarse models (Exp.2)





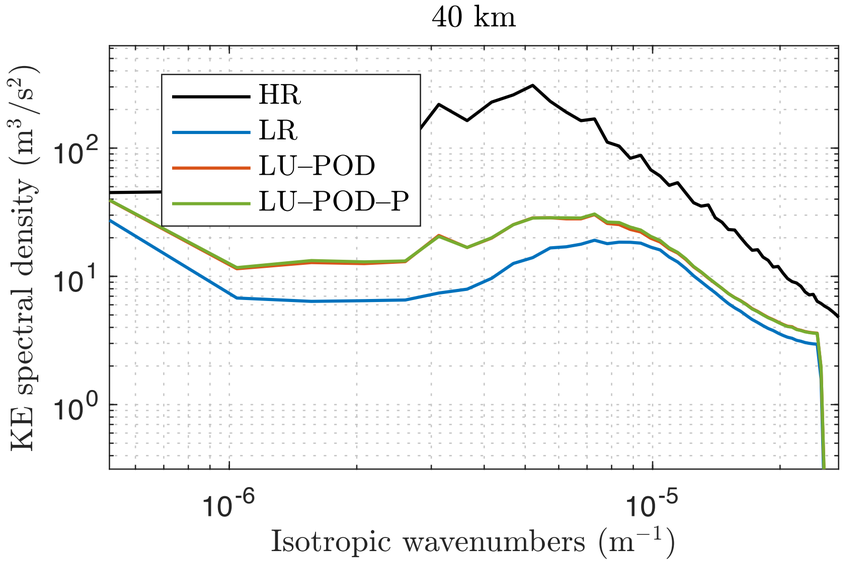


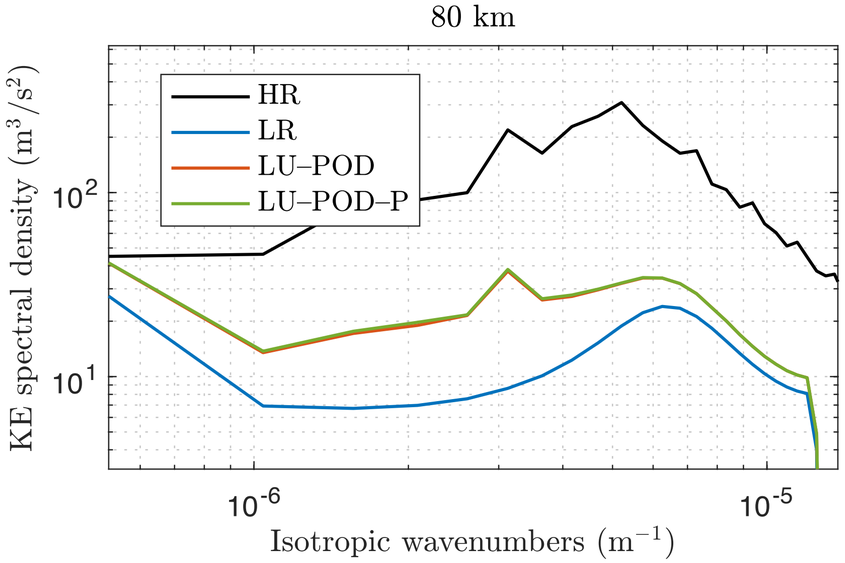




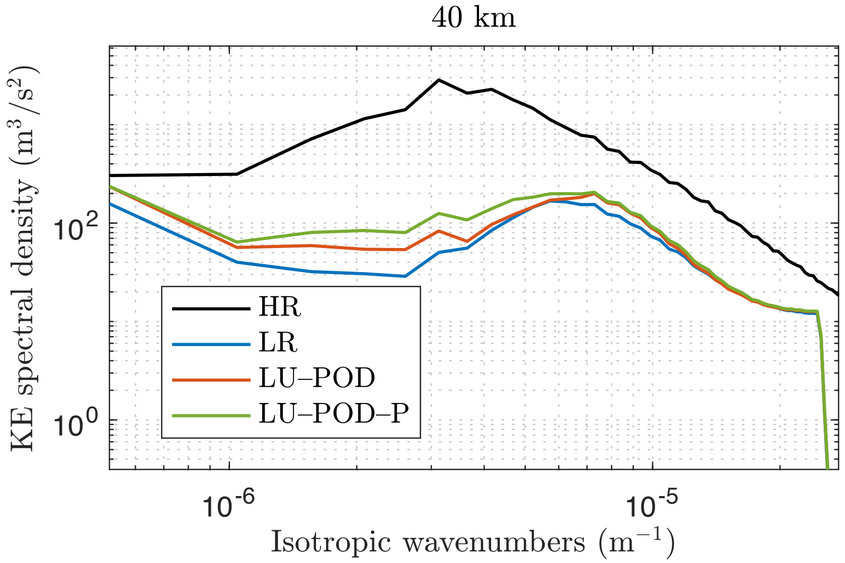
Energy analysis

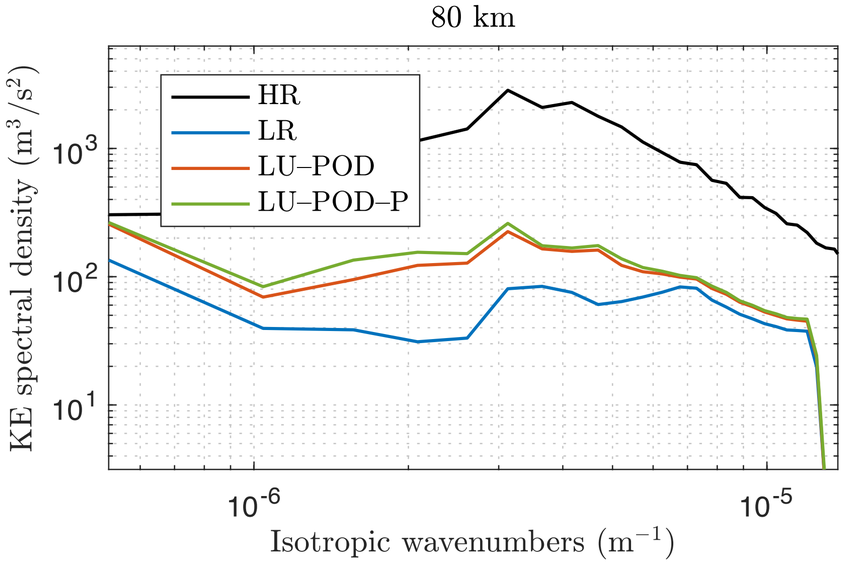
Exp.1 (without SST)





Exp.2 (with SST)





Comparison of KE spectral density averaged in time (60-75 yrs) and in layers for different coarse models

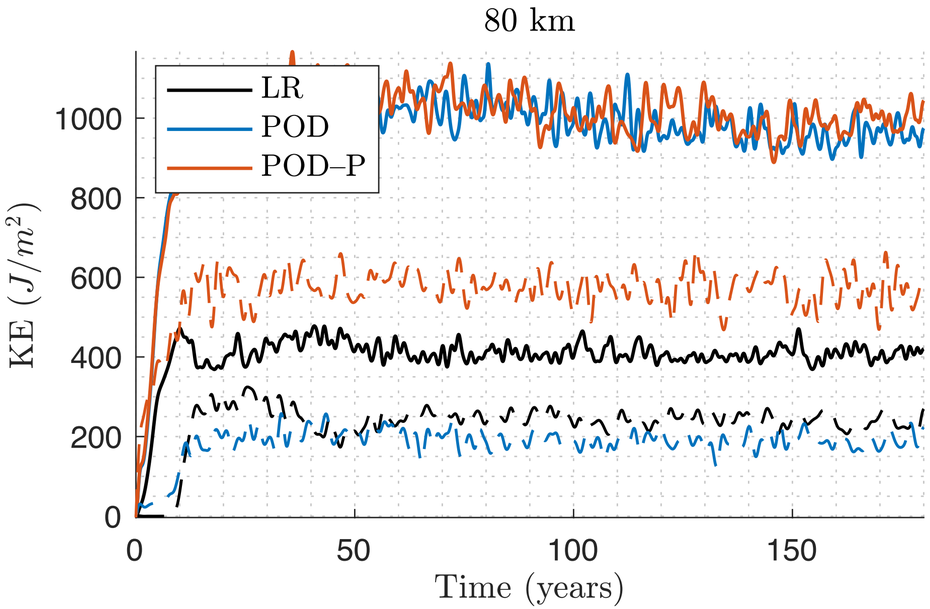
Energy analysis

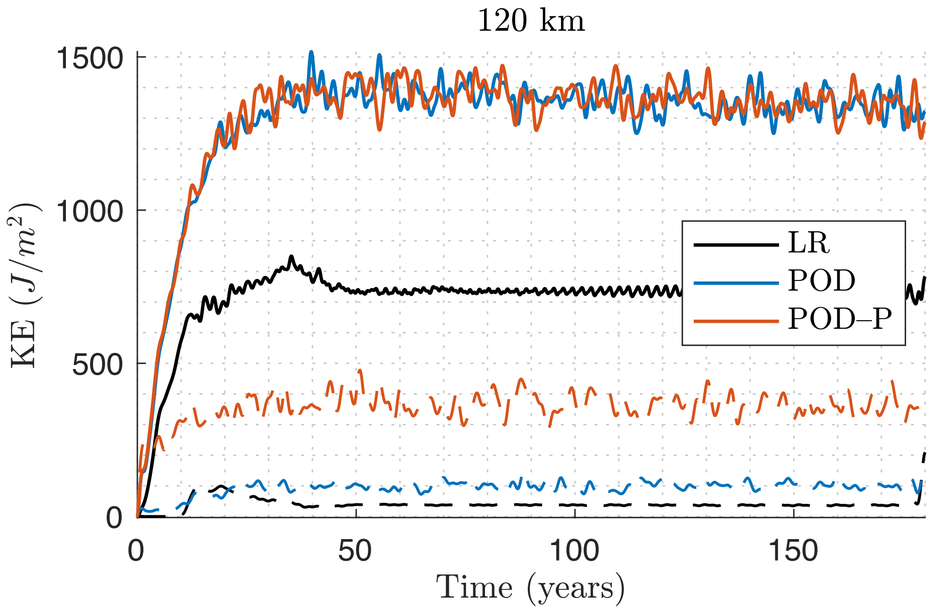
- KE decomposition

$$\boldsymbol{u}_k = \overline{\boldsymbol{u}}_k + \boldsymbol{u}'_k$$ **Low-pass filtering**

$$\mathrm{EKE}_k = \frac{\rho_0 H_k}{2} \|\overline{\boldsymbol{u}}_k\|_A^2 + \frac{\rho_0 H_k}{2} \|\boldsymbol{u}'_k\|_A^2$$

Standing EKE **Transient EKE**





Comparison of EKE (integrated over layers) for different coarse models (Solid lines — standing EKE, Dashed lines — transient EKE)

Energy analysis

- KE contributions

KE

PE

Buoyancy forcing

$$\partial_t \sum_{k=1}^{N} \frac{\rho_0 H_k}{2} \|\boldsymbol{u}_k\|_A^2 = -\partial_t \sum_{k=1}^{N-1} \frac{\rho_0 g'_k}{2} \|\eta_k\|_A^2 - \sum_{k=1}^{N-1} \rho_0 g'_k \langle \eta_k, w_k \rangle_A$$

(= 0 in Exp.1)

Wind forcing

$$+ \rho_0 \langle \boldsymbol{u}_1, \boldsymbol{\tau} \rangle_A - \frac{\rho_0 f_0 \delta_{\mathrm{ek}}}{2} \|\boldsymbol{u}_N\|_A^2$$

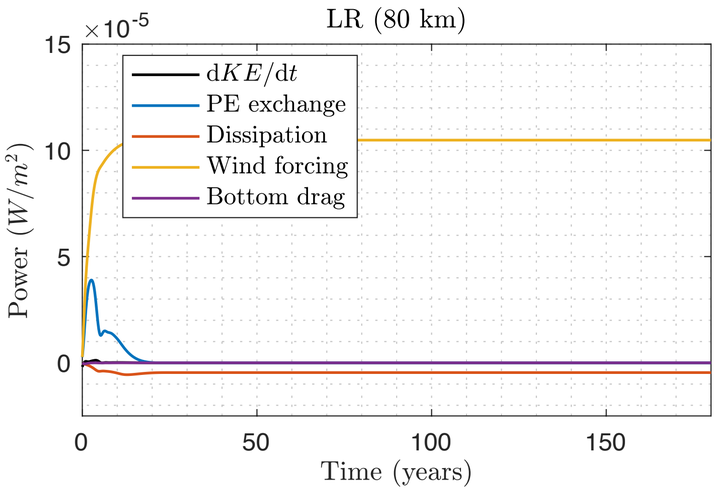
Bottom drag

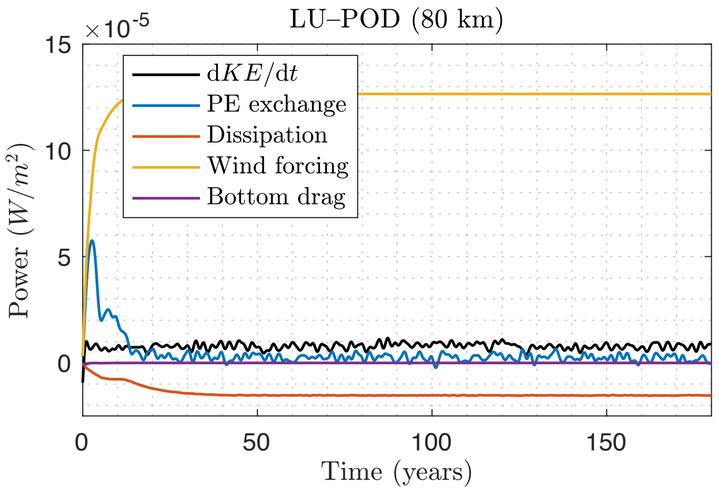
$$+ \sum_{k=1}^{N} \rho_0 H_k \langle \boldsymbol{u}_k, A_2 \nabla^2 \boldsymbol{u}_k - A_4 \nabla^4 \boldsymbol{u}_k \rangle_A$$

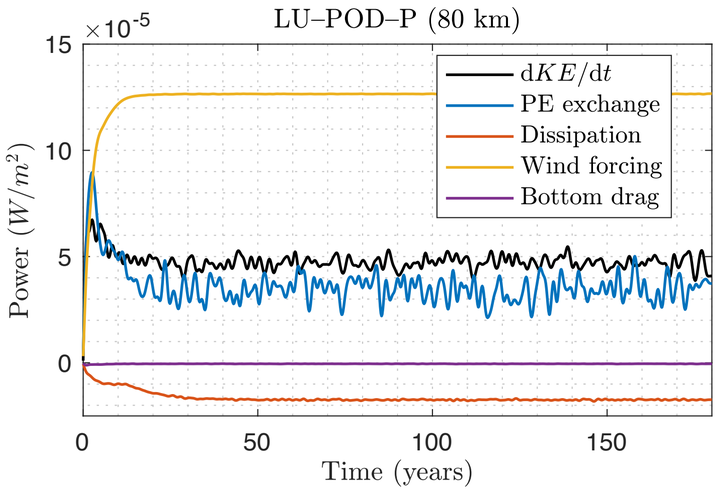
Dissipation

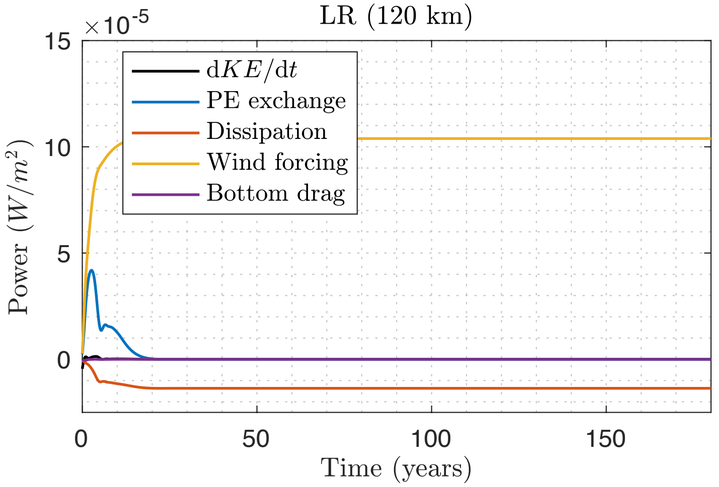
$$\langle f, g \rangle_A = \frac{1}{|A|} \int\int f g \, \mathrm{d}A$$

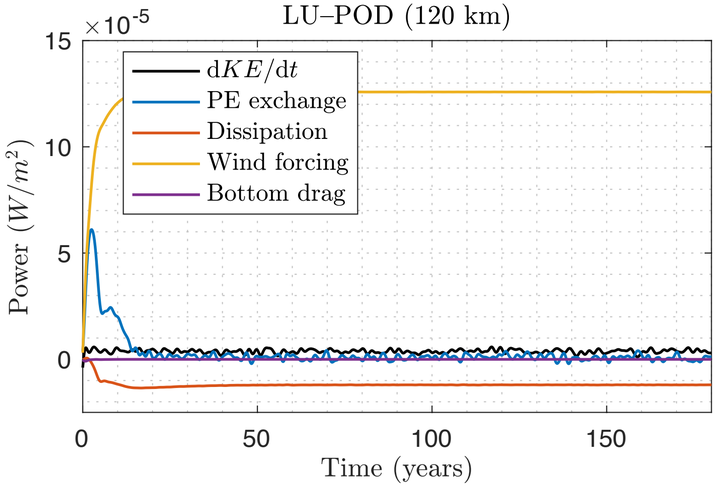
Energy analysis

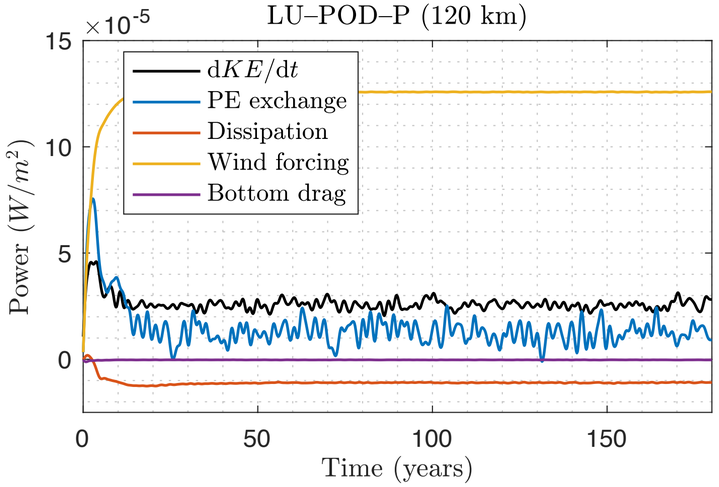






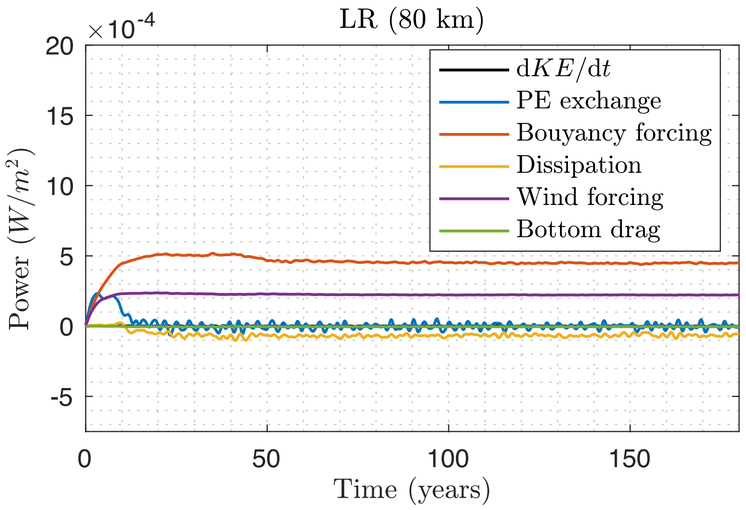


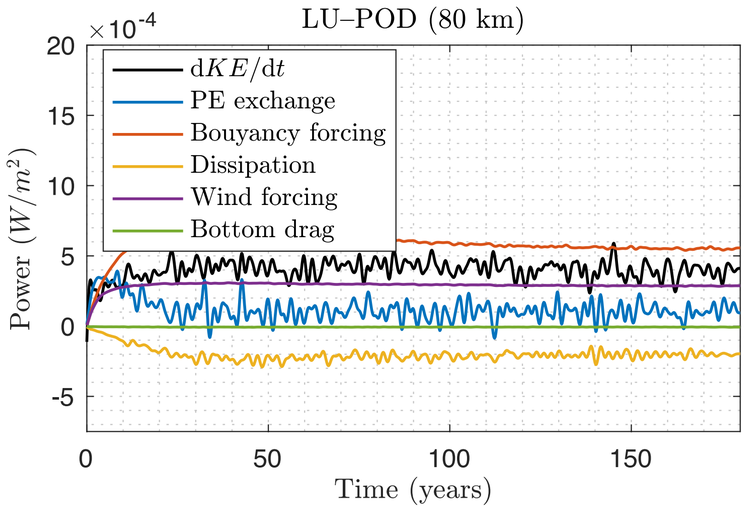


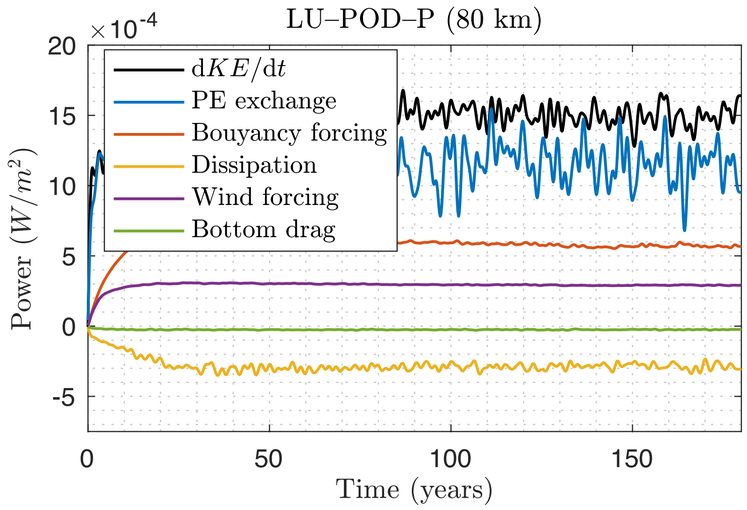


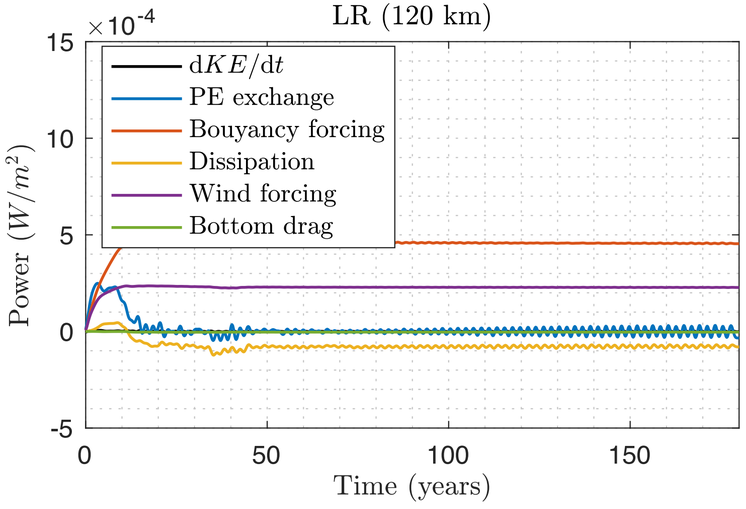
Comparison of contributions to the rate of KE for different coarse models (Exp.1)

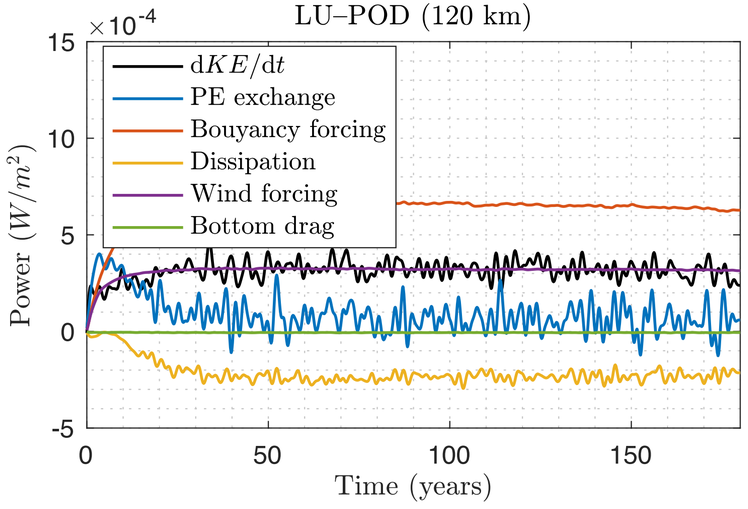
Energy analysis

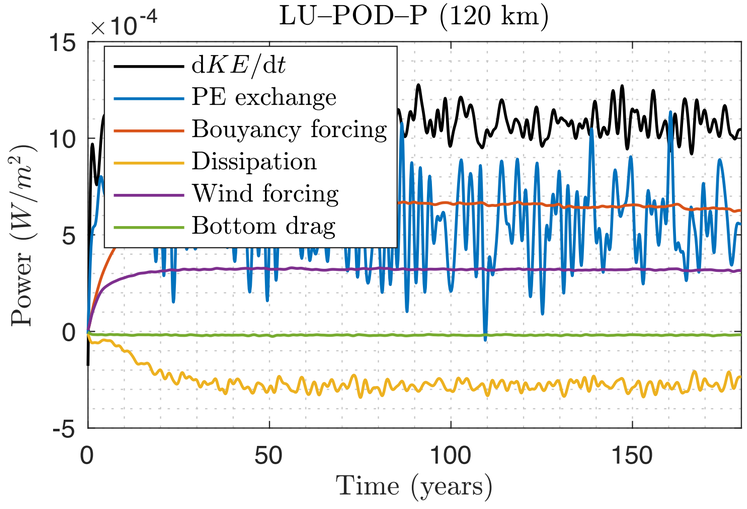












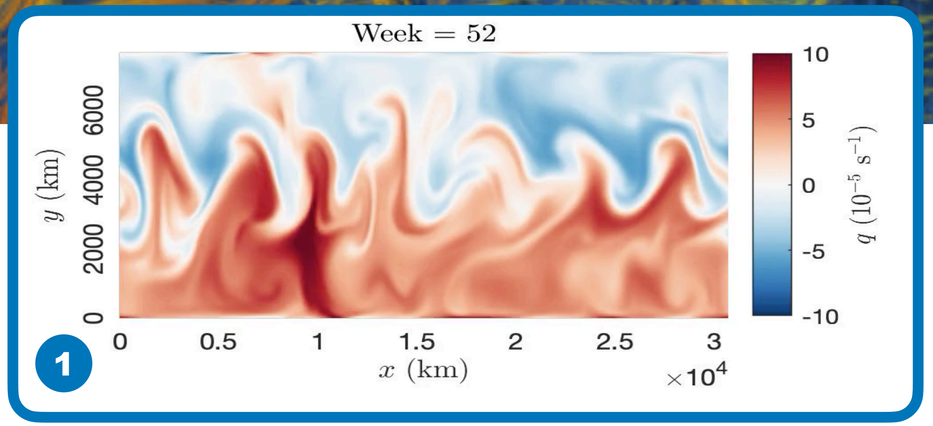
Comparison of contributions to the rate of KE for different coarse models (Exp.2)

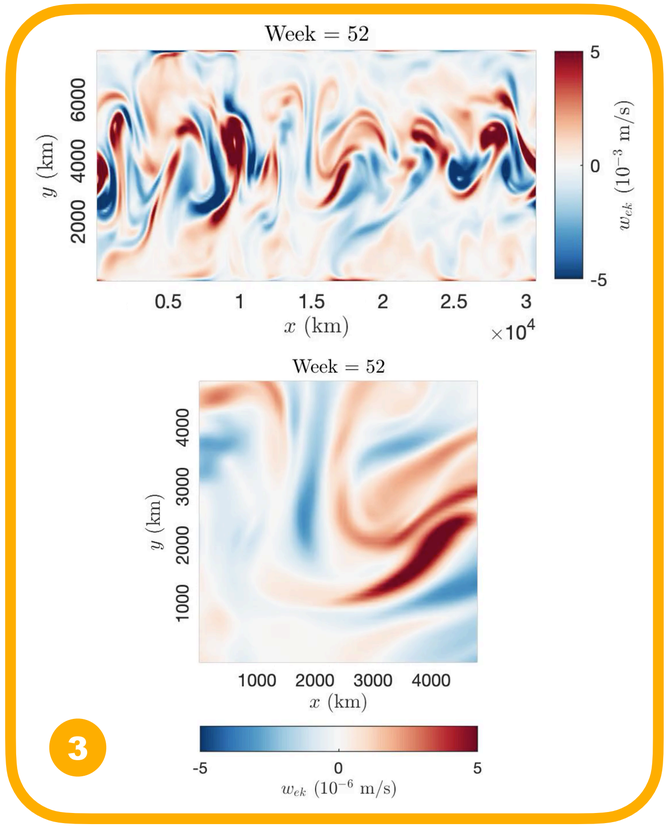
Conclusion

- The correction drift is found to be important in reproducing the meandering jet on coarse mesh.

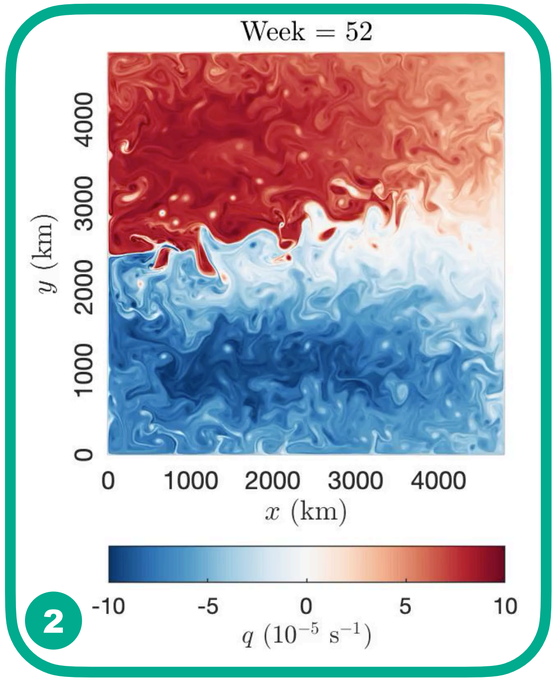
- The non-stationary noise based on the projection method enables us to improve the low-frequency variability of the large-scale circulation.

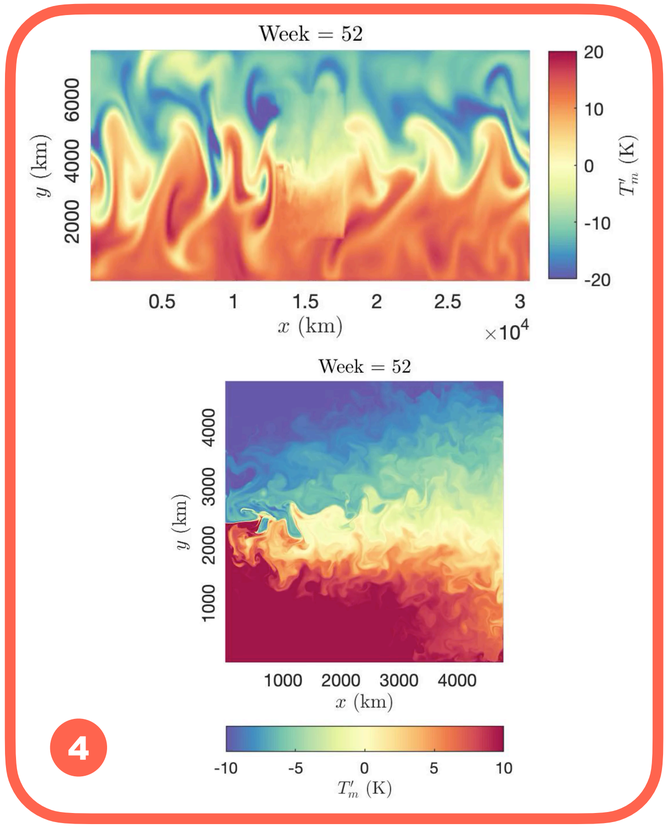
Working on …

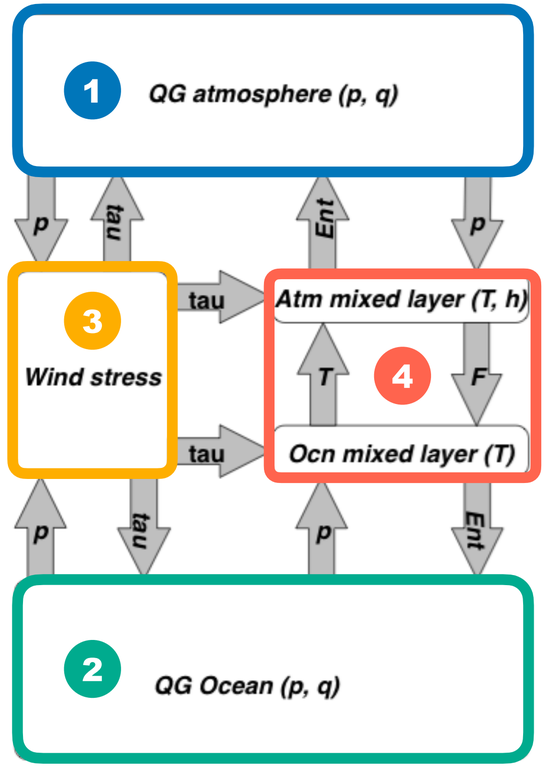




Coupled model








Thanks for your attention !